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1. (a) Using the macroscopic Maxwell equations to derive the boundary conditions for the electric field, $\vec{E}$, and the electric displacement, $\vec{D}$, at the interface of two media with dielectric constants of e_1 and e_2.
   (b) A very long, thin rod of dielectric constant e is placed in a homogeneous field $\vec{E}_0$, and is parallel to the direction of the field. What are the $\vec{E}$ and $\vec{D}$ in the interior of the rod.
2. Consider a point charge q inside a hollow, grounded, conducting sphere of inner radius a, and find.
   (a) the potential inside the sphere,
   (b) the induced surface-charge density, and
   (c) the magnitude and the direction of the force acting on q.
   What are the solutions of (a), (b), and (c) if the sphere is kept at a fixed potential V? What are the solutions of (a), (b), and (c) if the sphere has a total charge of Q, and is not grounded?
3. Two concentric spheres have radii a, $b(b>a)$, and are each divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential. Determine the potential in the region of $a \le r \le b$ as a series in Legendre polynomials. Include terms up to $l = 2$. Check your solution in the limiting cases of $b \to \infty$, and $a \to 0$.
4. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a (a<d) and of dielectric constant e. Find the potential at all points in space as an expansion in spherical harmonics.
5. Prove the uniqueness theorem for solutions of Poisson or Laplace quation for the electrostatic potential. Show that the electric field inside a cavity in a conductor vanishes.
6. If the charge density inside an atom can be described by
   $$r(r,q,f) = Ze(1/r^2)d(r)(\cos q)d(f) - Ze(h^3/8p)e^{-hg}(1 + a\sin q\cos f + b\cos q),$$
   Where Z and e are the atomic number and the elementary charge, respectively, and a and b are constants. Find the electrostatic potential of this atom.
7. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a (a<d) and dielectric constant e. Find the potential at all point in space as an expansion in spherical harmonics.
8. Using the **macroscopic** Maxwell equations for the magnetic field **H** and the magnetic induction **B** to derive the boundary conditions on **H** and **B** at the boundary of two mediums with magnetic permeability m_1 and m_2.
9. Derive the Maxwell displacement current and use Maxwell equations to obtain wave equations ofr the scalar potential Φ and the vector potential **A**.
10. A hollow right-circular cylinder of radius b has its axis coincident with the z axis and its ends at z = 0 and z = L. The potential on the end faces is zero, while the potential on the cylindrical surface is given as V(j , z). Using the appropriated separation of variables in cylindrical coordinates, find a series solution for the potential anywhere inside the cylinder.
    Note:the limiting forms of Bessel functions for small (kr) are given as the following:

$$r << 1 \qquad I_m(kr) \to \frac{1}{\Gamma(m+1)}\left(\frac{kr}{2}\right)^m \to 0$$

$$K_m(kr) \to -\left[\ln\left(\frac{kr}{2}\right)^m + 0.5772......\right] \to -\infty (m=0)$$

$$\to \frac{\Gamma(m)}{2}\left(\frac{2}{kr}\right)^m \to \infty (m \neq 0)$$

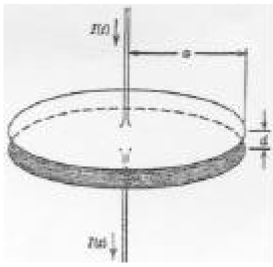
11. Two long, coaxial, cylindrical conducting surfaces of radii a and b are lowered vertically into a liquid dielectric. If the liquid rises an average height h between the electrodes when a potential difference V is established between them, show that the susceptibility of the liquid is

$$c_e = \frac{(b^2 - a^2) rgh \ln\left(\frac{b}{a}\right)}{e_0 V^2}$$

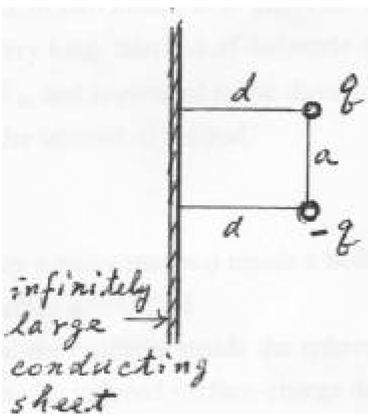
where r is the density of the liquid, g is the acceleration due to gravity, and the susceptibility of air is neglected.

12. An ideal circular parallel plate capacitor of radius a and plate separation d << a is connected to a current source by axial leads, as shown in the sketch. The current in the wire is I (t) = $I_0 \cos wt$.
    (a) Calculate the electric and magnetic fields between the plates to second order in powers of the frequency (or wave number), neglecting the effects of fringing fields.
    (b) Calculate the volume integrals of the harmonic electric magnetic energies W_e and W_m to second order in w .Show htat in termsof the input current I_i, define by $I_i = -iwQ$, where Q is the total charge on one plate, these energies are

$$\int w_e d^3x = \frac{1}{4pe_0}\frac{|I_i|^2 d}{w^2 a2}, \qquad \int w_m d^3x = \frac{m_0}{4p}\frac{|I_i|^2 d}{8c^2}\left(1 + \frac{w^2 a^2}{12c^2}\right)$$



13. Find the electrostatic potential in the whole space of the system shown in the figure.



14. Find the electrostatic potential of a charge distribution described by $r(r,q,f)=e^{-kr}(1+\sin q\cos f+\cos q)$. **Hint :**

$$\frac{1}{|\vec{r}-\vec{r}\,'|}=4p\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*(q',f')Y_{lm}(q,f)$$

15. Derive the boundary conditions for the electric field, $\vec{E}$, and displacement, $\vec{D}$, at the interface of two mediums with dielectric constants of e_1 and e_2.
16. Derive the boundary conditions for the magnetic induction, $\vec{B}$, and field, $\vec{H}$, at the interface of two magnetic materials with magnetic permeabilities of m_1 and m_2.
17. Derive Maxwell's displacement current and use Maxwell equations to obtain wave equations for the scalar and vector potentials.
18. Using the method of images, discuss the problem of a point charge q inside a hollow, grounded, conducting sphere of inner radius a. Find (a) the potential inside the sphere; (b) the induced surface-charge density; (c) the magnitude and direction of the force acting on q. (d) Is there any change in the solution if the sphere is kept at a fixed potential V? If the sphere has a total charge Q on its inner and outer surfaces?
19. A hollow, rectangular box with four sides at zero potential, while the fifth (z=c) has the specified potential V(x,y), and the sixth (y=b) has another specified potential U(x,z). Find the potential.
20. A straight-line charge with constant line charge density λ is located perpendicular to the x-y plane in the first quadrant at (x0,y0). The intersecting planes x = 0, y ≧0 and y = 0, x ≧ 0 are conducting boundary surfaces held at zero potential. Consider the potential, fields, and surface charges in the first quadrant. (a) The potential for an isolated line charge at (x0,y0) is
Φ(x,y) = (λ/4π ε 0)ln(R2/r2) ,R is a constant. Determine the expression for the potential of the line charge in the presence of the intersecting planes. Verify explicitly that the potential and the tangential electric field vanish on the boundary surfaces. (b) Determine the surface charge density on the plane y =0, x ≧ 0. (c) Show that the total charge (per unit length in z) on the plane y =0, x ≧ 0 is Qx = (-2λ/π) arctan(x0/y0). What is the total charge on the plane x =0.
21. How do the discontinuities in the electric field and potential originate from the surface distributions of charges and dipoles?
22. (a) In order to handle the boundary conditions in Poisson's equation, the mathematical tool, Green's Theorem has been developed. Describe it and give the solution of Poisson's equation. (b) How do the variation method derive the Poisson's equation? And outline the steps of variation method to solve the Poisson's equation.

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參考書本：Jackson

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第二章---習題 1、2、3、5、6、7、8、11、13、17、22、26

第三章---習題 1、2、5、6、7、14

第四章---習題 1、6、7、10

第五章---習題 1、2、3、6、11、13

第六章---習題 8

第七章---習題 3

第九章---習題 1、2